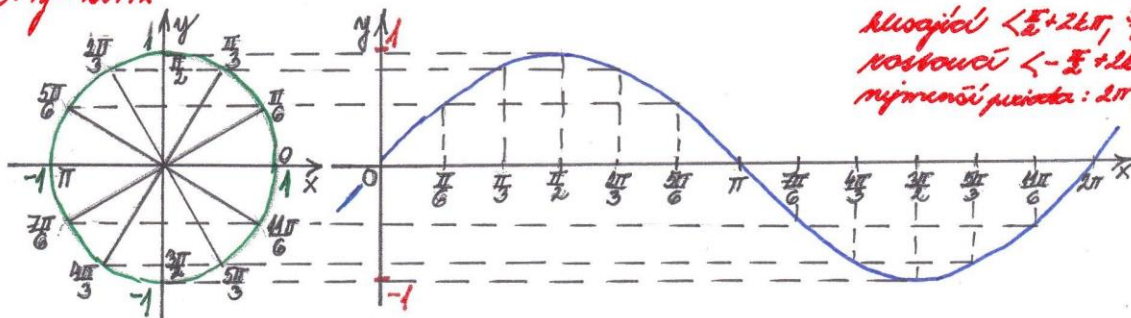


5. Grafy goniometrických funkcí

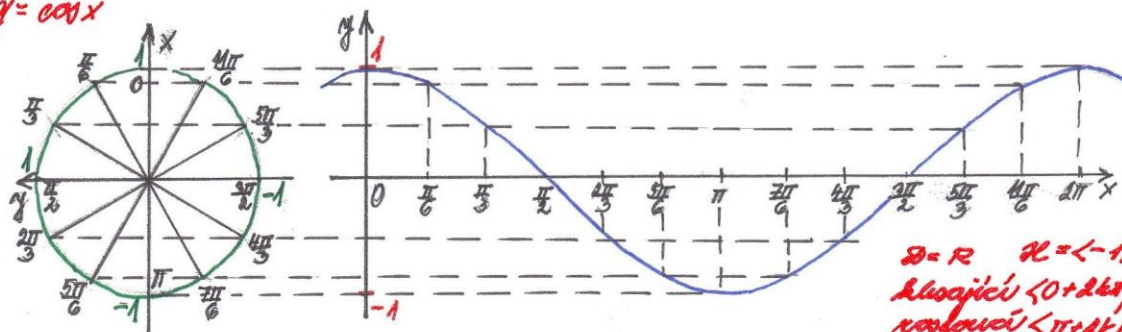
- pro sestavení grafů využijeme jednotkové kružnice
- pro grafy $y = \cos x$ a $y = \cotg x$ soustavu souřadnic otočnou o $90^\circ (\frac{\pi}{2})$
- pro srovnání odlišných hodnot

$f: y = \sin x$



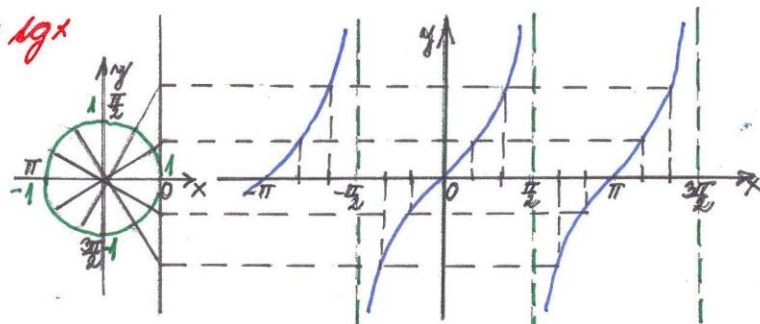
$D = \mathbb{R}$ $\mathcal{H} = \langle -1, 1 \rangle$ $\mathbb{R}$
 klesající $\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \rangle$
 rostoucí $\langle -\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi \rangle$
 nejmenší perioda: 2π (360°)

$f: y = \cos x$



$D = \mathbb{R}$ $\mathcal{H} = \langle -1, 1 \rangle$
 klesající $\langle 0 + 2k\pi, \pi + 2k\pi \rangle$
 rostoucí $\langle \pi + 2k\pi, 2\pi + 2k\pi \rangle$
 nebo $\langle -\pi + 2k\pi, 0 + 2k\pi \rangle$
 $\mathbb{R}$
 nejmenší perioda: 2π (360°)

$f: y = \tan x$

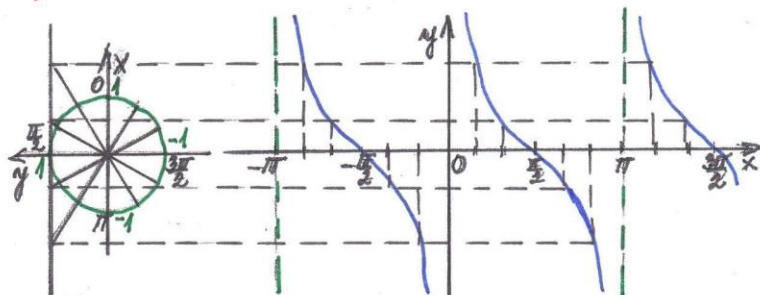


nedefinována pro LICHÉ
 NÁSOBKY $\frac{\pi}{2}$

$$D = \mathbb{R} - \bigcup_{k \in \mathbb{Z}} \left\{ \left(2k+1\right) \frac{\pi}{2} \right\}$$

$\mathcal{H} = \mathbb{R}$ $\mathbb{R}$
 rostoucí $\langle -\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi \rangle$
 nejmenší perioda: π (180°)

$f: y = \cotg x$



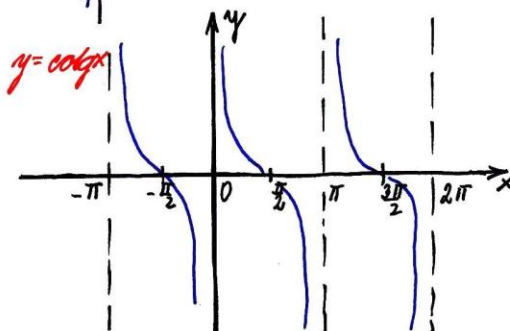
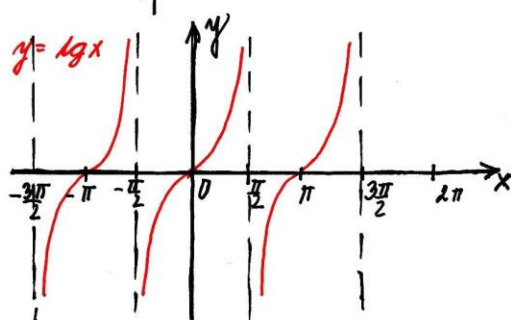
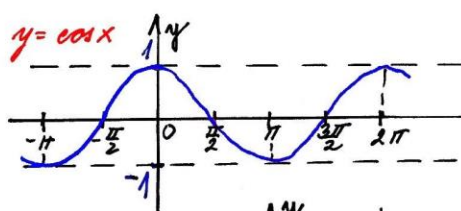
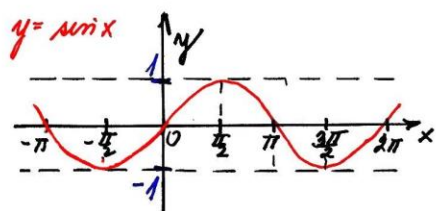
nedefinována pro
 NÁSOBKY π

$$D = \mathbb{R} - \bigcup_{k \in \mathbb{Z}} \{ k\pi \}$$

$\mathcal{H} = \mathbb{R}$ $\mathbb{R}$
 klesající $\langle -\pi + k\pi, 0 + k\pi \rangle$
 nebo $\langle 0 + k\pi, \pi + k\pi \rangle$
 nejmenší perioda: π (180°)

TABULKA HODNOT GONOMETRICKÝCH FUNKCÍ

	1. kv.	30°	45°	60°	90°	180°	270°	360°	-90°	-180°
	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$-\frac{\pi}{2}$	$-\pi$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1	0	-1
$\lg x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.	0	undef.	0	undef.	0
$\cotg x$	undef.	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	undef.	0	undef.	0	undef.



($\lg 0 = 0$, $\lg 90^\circ$ undef.)
 $\Rightarrow$ graf, perioda 180°

($\cotg 0$ undef., $\cotg 90^\circ = 0$)
 $\Rightarrow$ graf, perioda 180°

Příklady

① Vykreslete grafy funkcí

$f: y = \sin x$

$\mathcal{H}_f = \langle -1, 1 \rangle$

$g: y = 2 \sin x$

$\left[\text{pro } x = \frac{\pi}{2} (90^\circ) \Rightarrow y = 2 \sin 90^\circ \right]$

$y = 2$

$\Rightarrow$ 2x větší hodnoty y
 než pro $y = \sin x$

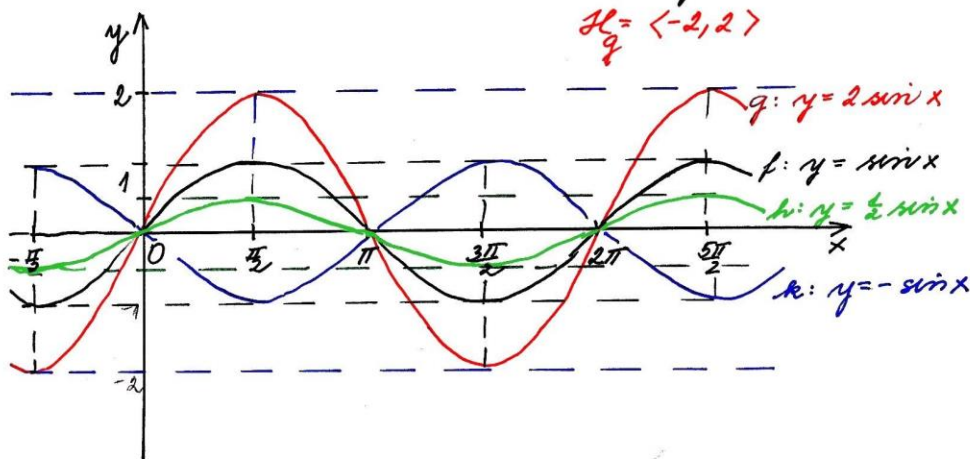
$\mathcal{H}_g = \langle -2, 2 \rangle$

$h: y = \frac{1}{2} \sin x (= \frac{\sin x}{2})$

$\left[x = \frac{\pi}{2} \Rightarrow y = \frac{1}{2} \sin \frac{\pi}{2} = \frac{1}{2} \right]$

$\Rightarrow$ poloviční hodnoty y
 než pro $y = \sin x$

$\mathcal{H}_h = \langle -\frac{1}{2}, \frac{1}{2} \rangle$



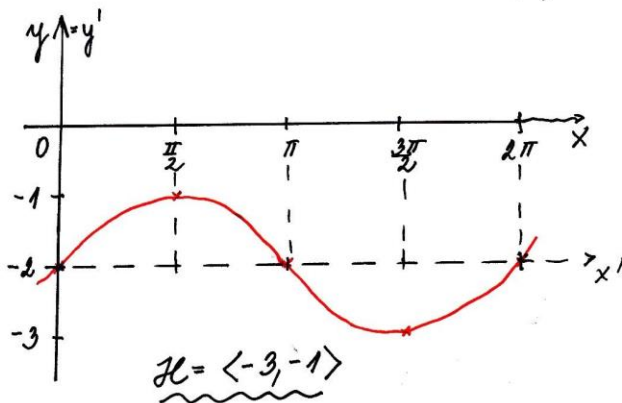
$k: y = -\sin x$

$\left[x = \frac{\pi}{2} \Rightarrow y = -\sin \frac{\pi}{2} \right]$
 $y = -1$

$\Rightarrow$ hodnoty s opačným
 znaménkem, stejná
 velikosti než $y = \sin x$

b) $f: y = \sin x - 2$
 $x_0 = 0, y_0 = -2$
 $[0; 2\pi]: y' = \sin x$

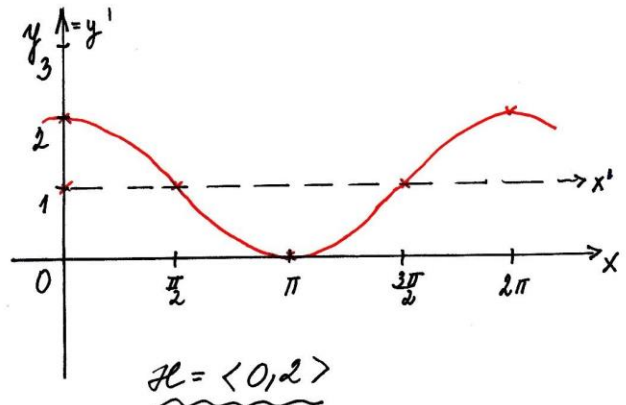
funkční hodnoty posunutých
 o 2 ke kmitům, kádrovní poloosa y'



typ: $y = \sin x - y_0$ $y = \cos x + 1$
 $x_0 = 0, y_0 = 1$

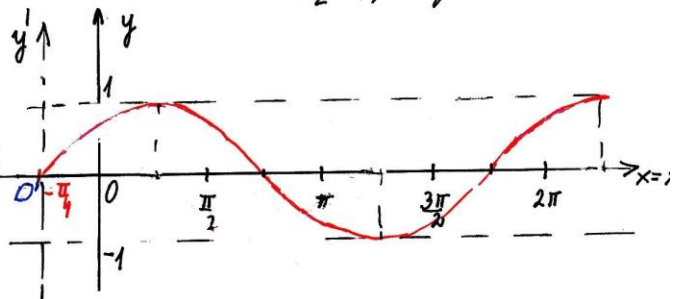
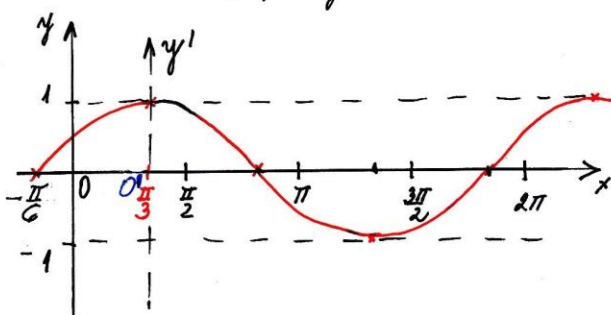
$[0; 2\pi]: y' = \cos x$

funkční hodnoty posunutých
 o 1 ke kmitům kádrovní poloosa y'



c) $f: y = \cos(x - \frac{\pi}{3})$ $typ: y = \sin(x + x_0)$
 $x_0 = \frac{\pi}{3}, y_0 = 0$
 $[0; 2\pi]: y' = \cos x$

$g: y = \sin(x + \frac{\pi}{4})$
 $x_0 = -\frac{\pi}{4}, y_0 = 0$
 $[0; 2\pi]: y' = \sin x$



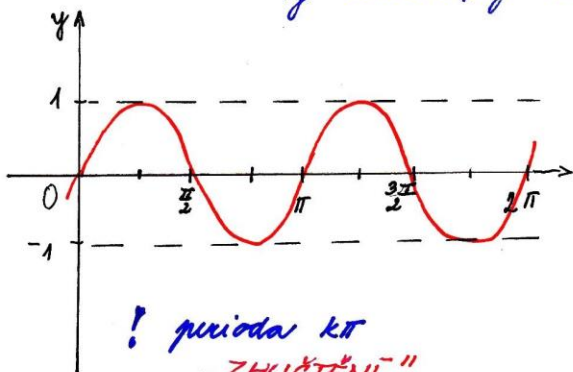
d) $f: y = \sin 2x$ $[x_0 = 0, y_0 = 0, o' [0, 0]]$ $typ: y = \sin bx$ $g: y = \cos \frac{x}{2}$ $[x_0 = 0, y_0 = 0, o' [0, 0]]$

! PERIODA 2x MENŠÍ než $y = \sin x$
 tj. perioda $k\pi$... "ZHUŠTĚNÍ"

$\sin 2x = \sin(2x + 2k\pi) \quad k \in \mathbb{Z}$
 $= \sin 2(x + k\pi)$

nějmenší perioda π
 perioda $k\pi, k \in \mathbb{Z}$

$y = \sin bx, y = \cos bx \Rightarrow$ nejmenší perioda $p = \frac{2\pi}{b}$



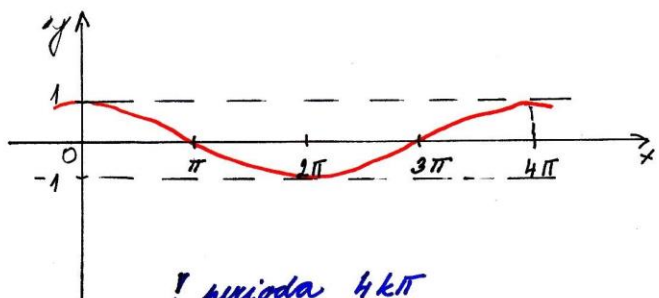
! perioda $k\pi$
 "ZHUŠTĚNÍ"
 nejmenší perioda π

! PERIODA 2x VĚTŠÍ než $y = \sin x$
 tj. perioda $2 \cdot 2k\pi = 4k\pi$

$\cos \frac{x}{2} = \cos(\frac{x}{2} + 2k\pi)$

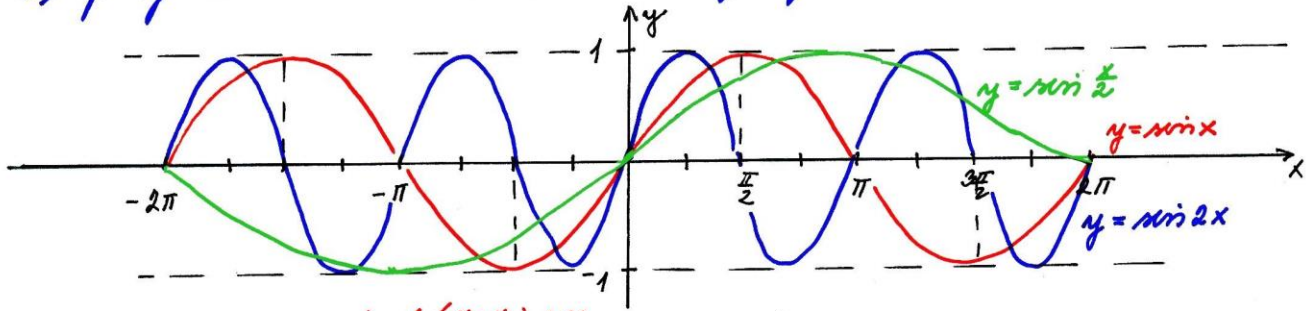
$\cos \frac{x}{2} = \cos \frac{1}{2}(x + 4k\pi)$

nějmenší perioda 4π
 perioda $4k\pi, k \in \mathbb{Z}$



! perioda $4k\pi$
 "ZŘEDĚNÍ"
 nejmenší perioda 4π

u) $f: y = \sin 2x \quad D = \langle -2\pi, 2\pi \rangle \quad g: y = \sin \frac{x}{2} \quad D = \langle -2\pi, 2\pi \rangle$



typ: $y = a \sin b(x - x_0) + y_0$

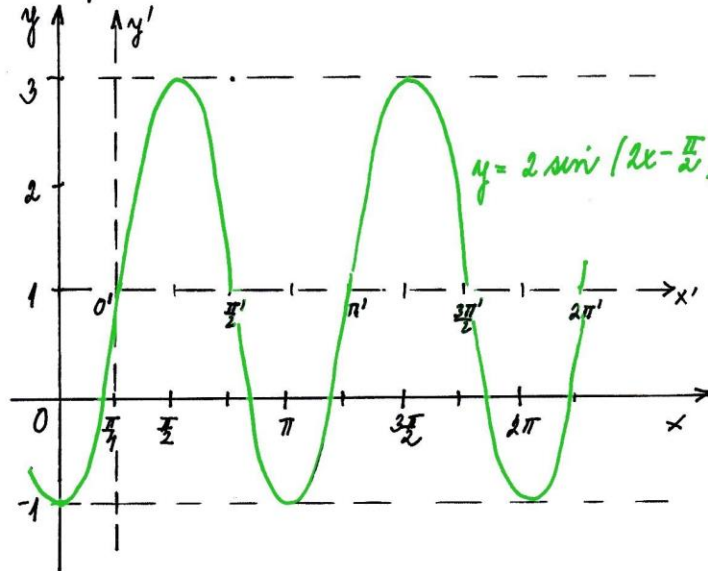
f) $y = 2 \sin(2x - \frac{\pi}{2}) + 1 = 2 \sin(2(x - \frac{\pi}{4})) + 1$

funkční
hodnoty
2x větší

$\left[\begin{array}{l} x_0 - \frac{\pi}{2} = 0 \\ x_0 = \frac{\pi}{2} \\ x_0 = \frac{\pi}{4} \end{array} \right. \quad y_0 = 1 \quad \left. \begin{array}{l} 0'[\frac{\pi}{4}, 1] \\ f': y = 2 \sin 2x \end{array} \right]$

perioda 2x menší, než $y = 2 \sin x$, y : perioda je $k\pi, k \in \mathbb{Z}$

než $y = \sin x$



obecně: $y = a \sin b(x - x_0) + y_0$

hodnoty y a klad větší
perioda 2x kratší
MENŠÍ
posun x o x_0
posun y o y_0
než $y = \sin x$

(v posunutí rovnostně nakreslení grafu pro $y = 2 \sin 2x$)

$D = \mathbb{R}$

$\mathcal{H} = \langle -1, 3 \rangle$

njm. perioda π

g) $y = \frac{1}{2} \cos \frac{1}{2}(x + \pi) - 1$

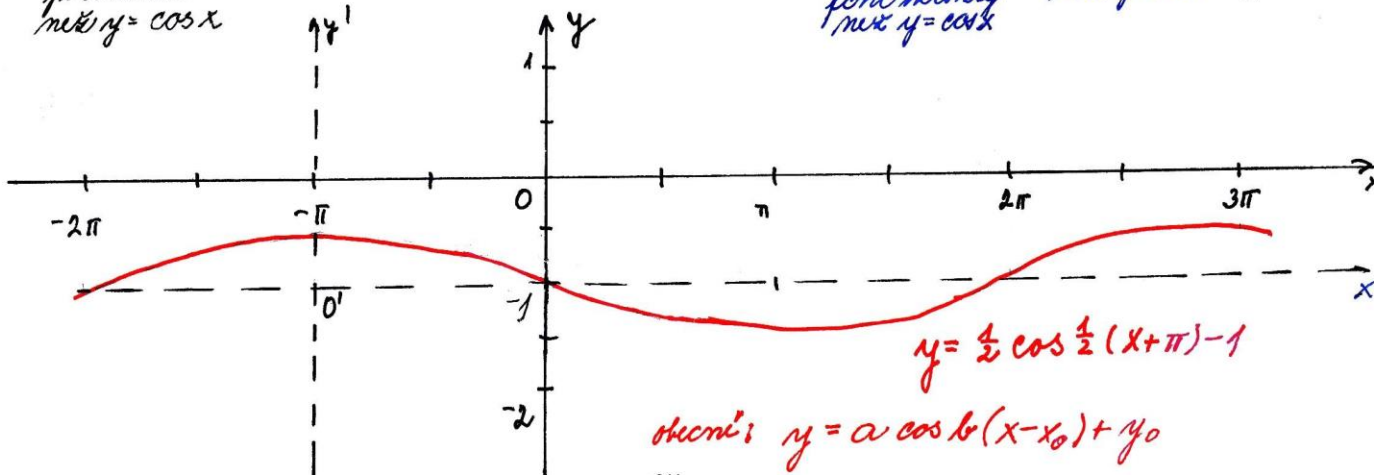
funkční
hodnoty
poloviční
než $y = \cos x$

njm. perioda
2x větší

$\left[\begin{array}{l} x_0 = -\pi \\ y_0 = -1 \end{array} \right. \quad \left. \begin{array}{l} 0'[-\pi, -1] \\ f': y = \frac{1}{2} \cos \frac{1}{2}x \end{array} \right]$

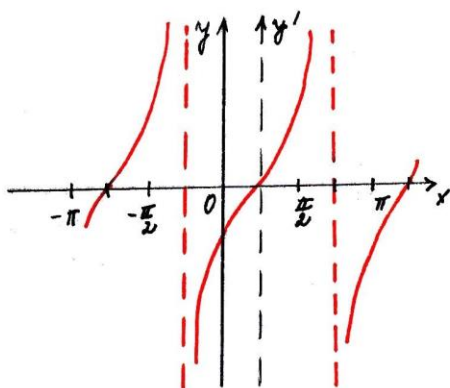
poloviční
funkční hodnoty
než $y = \cos x$

2x větší perioda
než $y = \cos x$

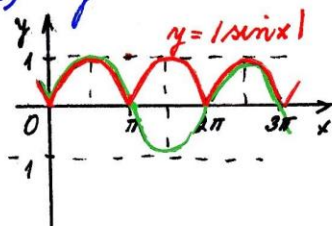


obecně: $y = a \cos b(x - x_0) + y_0$

h) $f: y = \lg(x - \frac{\pi}{4})$
 $[x_0 = \frac{\pi}{4}, y_0 = 0]$
 $O'[\frac{\pi}{4}, 0], y' = \lg x$

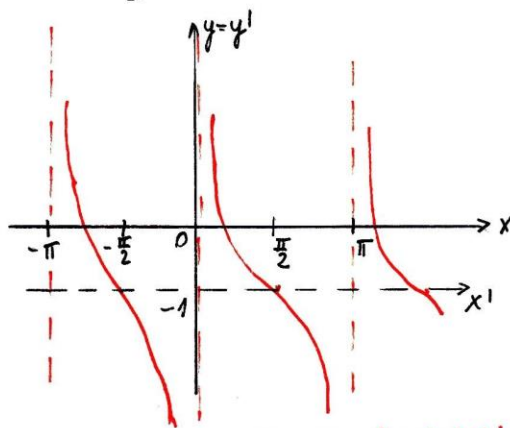


typ: $y = |\sin x|$
 ch) $y = |\sin x|$

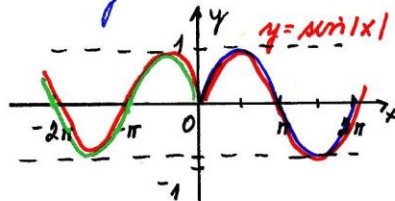


hodnoty fci
 $y = \sin x$ pod
 osou x resle.
 v osou soum.
 podle osy x

g: $y = \cotg x - 1$
 $[x_0 = 0, y_0 = -1]$
 $O'[0, -1], y' = \cotg x$

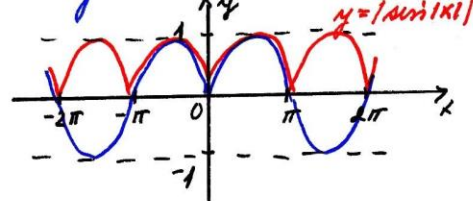


typ: $y = \sin|x|$
 $y = \sin|x|$



$y = |x| \dots$ suda fci (soum.
 podle osy y)
 1. $y = \sin x$ v $(0, +\infty)$
 2. v $(-\infty, 0)$ graf soum.
 podle osy y

typ: $y = |\sin|x||$
 $y = |\sin|x||$



1. graf fci $y = \sin|x|$
 2. napr. hodnoty v osou'
 soum. podle osy x

② Vypočítejte s využitím grafů gon. funkcí

du: a) $a^2 \sin \frac{\pi}{2} + b^2 \cos 0 + 2ab \cos \pi =$

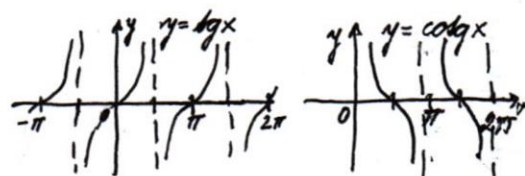
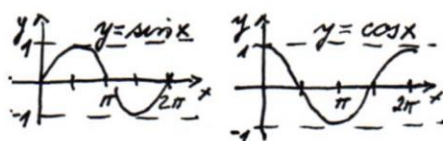
$= a^2 + b^2 - 2ab = (a-b)^2$

b) $3 \cos \frac{\pi}{2} - 4 \sin \frac{3\pi}{2} + 8 \lg \pi = -4 \cdot (-1) = 4$

c) $2 \cos \pi + 6 \cotg \frac{3\pi}{2} - 5 \sin 2\pi = -2$

d) $2 \lg 0 + \sin \pi - \cos \frac{3\pi}{2} - \cos \frac{\pi}{2} = 0$

NUTNÉ SI ČRTAT GRAFY!



③ Vypočítejte

du: a) $\lg 30^\circ \cdot \cotg 30^\circ - \sin 30^\circ \cdot \lg 60^\circ = \frac{\sqrt{3}}{3} \cdot \sqrt{3} - \frac{1}{2} \cdot \sqrt{3} = 1 - \frac{\sqrt{3}}{2}$

b) $\cotg \frac{12\pi}{9} \cdot \lg 11\pi \cdot \cotg \frac{19\pi}{3} \cdot \lg(-7\pi) =$
 $= \cotg \frac{4\pi}{3} \cdot \lg 0 \cdot \cotg \frac{\pi}{3} \cdot (-\lg 0) = 0$

c) $\frac{\lg(-\frac{\pi}{4}) \cdot \cotg(-\frac{\pi}{4})}{\sin(-\frac{3\pi}{2}) \cdot \cos(-4\pi)} = \frac{-\lg \frac{\pi}{4} \cdot (-\cotg \frac{\pi}{4})}{-\sin \frac{3\pi}{2} \cdot \cos 4\pi} = \frac{-1 \cdot (-1)}{-(-1) \cdot 1} = 1$

$\frac{12\pi}{9} = \frac{4\pi}{3}$ $\lg 11\pi = \lg(0 + 11\pi)$
 k tomu

$\frac{19\pi}{3} = \frac{\pi}{3} + 6\pi$
 k tomu

$\lg(-7\pi) = -\lg 7\pi =$
 $= -\lg(0 + 7\pi) = -\lg 0$
 perioda, k tomu